Answers:

1. $150°$.
2. 14
3. 8
4. 880
5. 11
6. $6\sqrt{7}$
7. 800π
8. $\frac{2\sqrt{6}}{3} + 2$ or $\frac{2\sqrt{6}+6}{3}$.
9. 28
10. 58
11. 5
12. $32\sqrt{2}$
13. 69
14. $22\sqrt{30}$
15. $2\sqrt{157}$
16. 66
17. 46
18. 262
19. $\frac{2\sqrt{3}}{3}$
20. 2
21. $75\sqrt{3}$
22. 200°
23. 280
24. $\frac{8\sqrt{5}}{3}$
25. $\frac{17}{8}$

1.) Since the number of diagonals for an n-gon is $\frac{n(n-3)}{2}$, then $\frac{n(n-3)}{2} = 54 \rightarrow n(n-3) = 108$ and observation is that 12 and 9 are two numbers with a product of 108 that differ by 3. Therefore, $n = 12$. Since all exterior angles (one at each vertex) have a sum of 360°, then the measure of each exterior angle is $\frac{360°}{12} = 30°$. Then each interior angle is $180° - 30° = \mathbf{150}°$.

2.) My goal will be to use Euler's formula (Faces + Vertices = Edges + 2), but I know only the number of faces is 12. However, 4 triangles will produce 12 sides, 4 rectangles produce 16 sides, and 4 pentagons produce 20 sides. This is a total of 48 sides, but when I "snap" them together to form the solid, I will always have two polygon sides snapping together to form one edge of the solid, so the solid will have $\frac{48}{2} = 24$ edges. Then Euler's formula becomes $12 + V = 24 + 2$ and we find $\boldsymbol{V = 14}$.

3.) First I graph the line $y = x + 4$ and I know the base is on this line. Then I know that the legs intersect at the origin. Then the altitude drawn to the base of this isosceles triangle will have one endpoint at the origin, and one endpoint at the midpoint of the segment connecting the x- and y-intercepts of $y = x + 4$, which gives us the point $(-2,2)$. This means the altitude has length $2\sqrt{2}$, and the base is bisected into parts of length 1. We now use the right triangles formed by dropping the altitude to find that the legs have length 3. So the perimeter is 3+3+2 or **8**.

4.) The radius of a particular bicycle tire is $\frac{3}{\pi}$ feet. How many revolutions will this tire make in a ride that is exactly one mile in length? The circumference of the circular bicycle tire is $C = 2\pi r = 2\pi\left(\frac{3}{\pi}\right) = 6$. Since the circumference is 6 feet, it will make $\frac{5280}{6}$ or $\mathbf{880}$ revolutions.

5.) It is helpful to have a healthy list of primitive Pythagorean triples in mind for this problem. 3-4-5 has 6 multiples that work, 5-12-13 has 2 multiples that work, 7-24-25 works, 8-15-17, and 20-21-29. This gives 6+2+1+1+1 or **11** triangles.

6.) I like working with a reduced triangle when possible, so I think of the sides being 4, 3, and $\frac{x}{6}$. I will multiply by 6 later to correct the reduction. Using the Pythagorean Theorem, I get $\frac{x}{6} = \sqrt{7}$, so $\boldsymbol{x = 6\sqrt{7}}$.

7.) The radius will be 10, and this along with the slant height of 26 leads us to find the height of the cone to be 24. So the volume of the cone is $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(10)^2(24) = \boldsymbol{800\pi}$.

8.) In this problem, visualization can be a roadblock. We essentially have a "triangular pyramid of balls" and the center of these spheres are the vertices of a regular tetrahedron whose edges are twice the radius of the ball, or 2 inches. Next, think about the fact that the base is an equilateral triangle with side length 2, then you can find the distance from the circumcenter of the triangle to the vertex is $\frac{2\sqrt{3}}{3}$. Now note that this circumradius along with the altitude of the tetrahedron and one edge of the tetrahedron form a right triangle with hypotenuse 2 and one leg $\frac{2\sqrt{3}}{3}$. Then the height of the tetrahedron is $\sqrt{(2)^2 - \left(\frac{2\sqrt{3}}{3}\right)^2} = \sqrt{4 - \frac{4}{3}} = \sqrt{\frac{8}{3}} = \frac{2\sqrt{6}}{3}$. The last step is to get down to the table from the tetrahedron, as well as up to the top of the top ball, which basically adds twice the radius length. So the top of the top ball is $\boldsymbol{\frac{2\sqrt{6}}{3} + 2}$ **or** $\boldsymbol{\frac{2\sqrt{6}+6}{3}}$.

9.) We know the angle formed by two secants, as one formed by a secant and a tangent, has a measure equal to half of the difference between the two intercepted arcs. In this case, the angle formed by the secants has measure x°, and this is half of the difference, so the whole difference is 2x°, which gives the missing upper left arc to be 3x°. A similar approach will show the bottom left arc to have measure 4x°. Now we have all of the arcs. So $x + 2x + 3x + 4x + 80 = 360$ and we get $\boldsymbol{x = 28}$.

10.) We might have been given the two legs, in which case the hypotenuse has length $\sqrt{\left(\sqrt{842}\right)^2 + \left(\sqrt{58}\right)^2} = \sqrt{842 + 58} = \sqrt{900} = 30$. We might have the hypotenuse and one leg, and the other leg is $\sqrt{\left(\sqrt{842}\right)^2 - \left(\sqrt{58}\right)^2} = \sqrt{842 - 58} = \sqrt{784} = 28$. To answer the question, we simply add 30 and 28 to get **58**.

11.) It is possible to have **5 pairs** of congruent parts without the triangles being congruent themselves. For example consider a triangle whose sides are $1, \sqrt{2}, and\ 2$. A similar triangle might have sides $\sqrt{2}, 2, and\ 2\sqrt{2}$. (The sides just need to be a geometric sequence where the ratio is not 1, and lies somewhere strictly between the golden ratio and its reciprocal in order to satisfy the triangle inequality theorem.)

12.) There are at least two approaches here. If you know the sine value for 45°, then simply divide the octagon into 8 triangles by drawing the circumradii, which will have $\frac{360°}{8}$ $or\ 45°$ between them. The area of each triangle can be found by the formula $A = \frac{1}{2}absinC = \frac{1}{2}(4)(4)\sin 45° = \left(\frac{1}{2}\right)(4)(4)\left(\frac{\sqrt{2}}{2}\right) = 4\sqrt{2}$. Then the 8 triangles together have area $\mathbf{32\sqrt{2}}$. The other approach would be to do it without trig. Look at the diagram below, which emphasizes one of the triangles formed by a side and the two bounding circumradii. When an altitude is dropped to one of the legs, a 45-45-90 triangle is formed with hypotenuse of 4, giving the legs, and the height of the triangle to be $2\sqrt{2}$. Then the area of the original triangle is $\frac{1}{2}bh = \frac{1}{2}(4)\left(2\sqrt{2}\right) = 4\sqrt{2}$ which leads to the same value of $\mathbf{32\sqrt{2}}$ for the octagon area.

13.) Using the triangle congruence, we know $\angle F \cong \angle C$, so $m\angle C = 5x + 5$. Then $m\angle A + m\angle B + m\angle C = 4x + 13 + 36 + 5x + 5 = 180$. This gives x = 14, and gives $m\angle D = m\angle A = \mathbf{69}$.

14.) The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into segments of length 10 and 12. What is the area of this right triangle? Note that the altitude to the hypotenuse of a right triangle is the geometric mean between the pieces of the hypotenuse. So the height is $\sqrt{(10)(12)} = 2\sqrt{30}$. Then, using the hypotenuse as the base, we have the area of the triangle is $\frac{1}{2}bh = \frac{1}{2}(10 + 12)\left(2\sqrt{30}\right) = \mathbf{22\sqrt{30}}$.

15.) The greatest possible distance between the points will come when the segment between the two circle centers (-7, 5) and (4, -1) is extended until it hits the two circles, which will have a length equal to the distance between the centers plus each radius, we'll say $d + r_1 + r_2$. The minimum length will be along the same line but will be $d - r_1 - r_2$. The sum of these two distances will be simply $2d = 2\sqrt{(4 + 7)^2 + (-1 - 5)^2} = 2\sqrt{(11)^2 + (-6)^2} = 2\sqrt{121 + 36} = \mathbf{2\sqrt{157}}$**.**

16.) The area of the kite is $\frac{1}{2}d_1 d_2$, and we know one diagonal has length 5+16 or 21. So $\frac{1}{2}(21)(d_2) = 252$. This leads us to the other diagonal having length 24. The diagonals of a kite are perpendicular, and one is bisected, which would have to be the one of length 24. So we end up with two right triangles with legs of length 5 and 12, which produces two sides of the kite having length 13. The other two right triangles have legs 12 and 16, which produces sides of the kite equal to 20. So the perimeter of the kite is 13+13+20+20 or **66**.

17.) Quadrilateral ABCD is inscribed in circle P. If the radius of circle P is 5 in., $m\angle A = 10x + 2, m\angle B = 8x + 3, m\angle C = 68$ and $m\angle D = 6x + 23$. Find the positive difference between the degree measures of arc AC and arc BD. First of all, opposite angles of an inscribed quadrilateral are supplementary. Using this fact, we can get x = 11 and $m\angle A = 112$,

$m\angle B = 91$, $m\angle C = 68$, and $m\angle D = 89$. Then I sketch the diagram below, letting the measures of arcs AB and CD be *a* and *b*, respectively. Since the angles are inscribed, the intercepted arc measures are twice the measure of the inscribed angle. The measures of arcs ABC and BCD are 178° and 224°, respectively. This gives $a + c = 178$ and $b + c = 224$. Subtracting the first from the second gives $b - a = \mathbf{46}$.

18.) A right rectangular prism has faces with areas of 35 and 56. If the volume of the solid is 280, what is the surface area of this prism? Let the dimensions be x, y, and z. Then there are separate faces with areas xy, xz, and yz. If we multiply these together, we get $(xy)(xz)(yz) = x^2y^2z^2 = V^2$. Since we know two face areas, the equation becomes $(35)(56)(yz) = 280^2$. So $yz = \frac{280\cdot 280}{35\cdot 56} = 40$. The surface are is $2(35 + 56 + 40) = 2(131) = \mathbf{262}$.

19.) A 45°-45°-90° triangle and 30°-60°-90° triangle share a hypotenuse. What is the ratio of the of the larger triangle area to the smaller triangle area? If you let the shared hypotenuse have length 2, you will get the legs of the 45°-45°-90° triangle each to be $\sqrt{2}$ and the legs of the 30°-60°-90° triangle to be 1 and $\sqrt{3}$. This gives areas of 1 and $\frac{\sqrt{3}}{2}$. The ratio of the larger to the smaller is $\frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} = \mathbf{\frac{2\sqrt{3}}{3}}$.

20.) A quick sketch will help. We don't know yet whether the center is inside or outside the trapezoid, but this will be evident by the sign of our answer for *x* in the end. We can drop an altitude on each side of the trapezoid from the ends of the top base to the longer base. We can realize then that we have 6-8-10 right triangles. Then, drawing the trapezoid altitude that (potentially) passes through center of the circle, as well as the two radii from the center to two of the vertices, we can see right triangles. The Pythagorean Theorem yields $x^2 + 11^2 = r^2$ and $(8 - x)^2 + 5^2 = r^2$. Substitution yields $x^2 + 11^2 = (8 - x)^2 + 5^2$, and solving gives $x = -2$. The negative just means that our drawing is a little off, and that the center actually lies outside the trapezoid. However, the center is still **2 units** from the longer base.

21.) A regular hexagon has side length 10. What is the area of the largest triangle which will fit inside this hexagon? The regular hexagon is made up of 6 equilateral triangles with side length 10. Each equilateral triangle will have area $25\sqrt{3}$, so the area of the hexagon is $150\sqrt{3}$. The largest eq. triangle that will fit inside the hexagon is formed by connecting every other vertex. The area of this triangle is half of the area of the hexagon. If you need to convince yourself, draw the hexagon, the triangle, and the radii of the triangle. You can see three rhombuses (or rhombi), the areas of which are split between the interior and exterior of the triangle. The area of the equilateral triangle is $\mathbf{75\sqrt{3}}$.

22.) The supplement of an angle is 20 degrees less than three times the complement of the angle. Find the sum of the complement and the supplement of this angle? The supplement and complement of an angle x can be expressed as $180 - x$ and $90 - x$, respectively. Then the equation is simply $180 - x = 3(90 - x) - 20$ which gives $x = 35$. The supplement is 145°, the complement is 55°, and their sum is **200°.**

23.) Ali runs around a $\frac{1}{4}$-mile track at a constant rate of 2 minutes per lap. Benson runs the same track at a constant rate of 3.5 minutes per lap. How many seconds will it take Ali to catch Benson from behind (or "lap him") if they both start at the same place at the same time? This means Ali runs mile in 8 minutes and Benson runs a mile in 14 minutes. Their rates are then $\frac{15}{2}$ and $\frac{30}{7}$, respectively. We will use the equation $d = r \cdot t$, and we want Ali to travel $\frac{1}{4}$ mile more than Benson. So $\frac{15}{2} \cdot t = \frac{30}{7} \cdot t + \frac{1}{4}$. Multiplying by 28 gives the equation $210t = 120t + 7$ and

$t = \frac{7}{90}$. But our answer is in hours, and we want seconds. Since there are 3600 seconds in each hour, $t = \frac{7}{90} \cdot 3600 = \mathbf{280}$ **seconds**.

24.) What is the length of the altitude drawn to the longest side of a triangle whose sides are length 7, 8, and 9? Using Heron's formula, you get the area of the triangle is $12\sqrt{5}$. We want the altitude drawn to the longest side, so $A = \frac{1}{2}bh \to 12\sqrt{5} = \frac{1}{2}(9)(h)$ giving $\boldsymbol{h} = \frac{\mathbf{8\sqrt{5}}}{\mathbf{3}}$.

25.) The surface area of a cone is 200π. If the sum of the radius and slant height is 25, what is the ratio of the lateral area to the base area? Surface area of a cone is given by $SA = \pi r^2 + \pi rl = \pi r(r+l)$. Since we know the SA and $r + l$, the equation is $200\pi = \pi r(25)$ which gives $r = 8$ and $l = 17$. Then the lateral area is $\pi rl = \pi(8)(17) = 136\pi$ and the base area is $\pi r^2 = \pi 8^2 = 64\pi$. The ratio of LA to BA is then $\frac{136\pi}{64\pi} = \frac{\mathbf{17}}{\mathbf{8}}$. (It might also be noted that the surface area, when written as LA plus BA simply divides it into the ratio of the radius and slant height.)